A Simple Friction Pressure Drop Correlation for Two-Phase Flow in Pipes

Eine einfache Gleichung zur Berechnung des Reibungsdruckverlustes bei der Zweiphasenströmung in Rohren

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Abstract

A new correlation for the prediction of frictional pressure drop for two-phase flow in pipes is suggested which is simple and more convenient to use than other methods. To determine their reliabilities, this correlation and fourteen correlations from the literature were checked against a data bank containing 9300 measurements of frictional pressure drop for a variety of fluids and flow conditions. It was found that the best agreement between predicted and measured values was obtained using the correlation suggested by Bandel. Somewhat less but still reasonable accuracy of pressure drop prediction is provided by a group of identified correlations, which includes the correlation described in this paper.

Kurzfassung

Es wird eine neue Gleichung zur Berechnung des Reibungsdruckverlustes bei der Gas–Flüssigkeitsströmung in Rohren vorgestellt. Diese Korrelation ist wesentlich einfacher als bisher publizierte Rechenverfahren und enthält nur zwei Anpassungsparameter, von denen der eine den Wert 2 und der andere den Wert 3 hat. Mit der neuen Korrelation und mit 14 Korrelationen anderer Autoren berechnete Werte wurden mit etwa 9300 Messwerten des Reibungsdruckverlustes verschiedener Gas–Flüssigkeitsströmungen verglichen. Dabei wurde festgestellt, dass die von Bandel vorgeschlagene Korrelation die beste Übereinstimmung zwischen berechneten und gemessenen Werten ergibt. Es folgt eine Gruppe von Korrelationen, zu der auch die in dieser Arbeit vorgeschlagene Korrelation gehört, die eine immer noch brauchbare Voraussage des Reibungsdruckverlustes ermöglichen. Abweichungen zwischen berechneten und gemessenen Reibungsdruckverlusten von durchschnittlich über 30% müssen beim heutigen Stand des Wissens immer noch akzeptiert werden.

Synopse

In der vorliegenden Arbeit wird ein äusserst einfaches Verfahren zur Berechnung des Reibungsdruckverlustes bei der Zweiphasen-Gas–Flüssigkeitsströmung in Rohren beschrieben. Die Grundlage für diese Korrelation war die Beobachtung, dass der Reibungsdruckverlust bei einem Dampfgehalt von $\dot{x} = 0.5$ etwa gleich dem Reibungsdruckverlust der reinen Dampfströmung ($\dot{x} = 1$) bei gleicher Gesamtmassenstromdichte ist. Darauf aufbauend wird in den Gl. (1)–(9) eine Korrelation entwickelt. Neben ihrer Einfachheit hat die vorgeschlagene Korrelation auch den Vorteil, dass sie leicht integriert werden kann, wenn das Verdampferrohr mit konstanter Wärmestromdichte beheizt wird. In diesem Fall muss dann der Beschleunigungsdruckverlust zu dem berechneten Reibungsdruckverlust addiert werden [7]. Die Abb. 2–4 zeigen einen Vergleich von gemessenen und mit der neuen Korrelation berechneten Einflüssen des Dampfgehaltes $\dot{x}$, der Massenstromdichte $\dot{m}$ und des reduzierten Druckes $p_r = p/p_c$ auf den Reibungsdruckverlust. In allen drei Abbildungen erkennt man, dass die Übereinstimmung zwischen berechneten und gemessenen Werten zufriedenstellend ist. Die in dieser Arbeit empfohlene Korrelation wurde weiterhin mit den in Tabelle 1 angegebenen Korrelationen aus der Literatur verglichen. Wie Bild 5 anhand von Messungen mit Stickstoff zeigt, wird mit einigen dieser Korrelationen (Nr. 2, Nr. 3, Nr. 4) der Einfluss des Dampfgehaltes auf den Reibungsdruckverlust nicht richtig beschrieben. Andere, zum Beispiel Nr. 2, Nr. 3, Nr. 4, Nr. 10, Nr. 11 und Nr. 14 schliessen den Reibungsdruckverlust der reinen Flüssigkeits- bzw. Gasströmung nicht mit ein. Die Korrelation von Bandel, Nr. 1, kann nur im Bereich $0.001 < \dot{x} < 0.98$ verwendet werden. Da die Korrelationen Nr. 6 und Nr. 7 auf dem

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homogenen Modell basieren, erhält man mit ihnen bei höheren Dampfgehalten zu niedrige Werte des Reibungsdruckverlustes. An der Schnittstelle zwischen den Korrelationen von Kesper [21] und Moussalli [22] können sich bei der Korrelation Nr. 10 Unstetigkeiten ergeben. Die in dieser Arbeit vorgeschlagenen Korrelation kann im gesamten Dampfgehaltsbereich $0 \leq \dot{x} \leq 1$ verwendet werden.

Die Vorhersagen der insgesamt 15 Korrelationen wurden mit Messwerten des Druckverlustes bei der Zweiphasenströmung verschiedenster Fluide verglichen. Vom Vergleich ausgeschlossen wurden alle Messwerte, die unter $dp/dL = 20\ N\ m^{-3}$ lagen, da in diesem Bereich grosse Streuungen zwischen den Messungen selbst vorlagen. Weiterhin wurden nur Messungen des reinen Reibungsdruckverlustes verwendet, um durch die Anwendung von Korrelationen für den Dampfvolumenanteil auftretende Unsicherheiten zu vermeiden. Übrig blieben schliesslich 9313 Messwerte der in den Tabellen A-1 und A-2 angegebenen Autoren. Eine Beschreibung der verwendeten Datenbank findet man im Anhang A an diese Arbeit.

Als charakteristische Werte für die Übereinstimmung zwischen berechneten und gemessenen Werten wurden der mittlere relative Fehler (Gl. (14)), der mittlere absolute Fehler (Gl. (15)) und der Prozentsatz der Daten mit einem relativen Fehler unter ±10% und ±20% und ±30% bestimmt. Tabelle 2 enthält einen Vergleich der verschiedenen Korrelationen im Hinblick auf ihre Übereinstimmung mit der Datenbank. Es zeigt sich, dass die Korrelation von Bandel [5], die aufgrund der bereits erwähnten Einschränkung nur mit 8541 Messdaten verglichen werden konnte, allen anderen untersuchten Korrelationen überlegen ist. Recht ordentliche Ergebnisse erhält man auch mit den Korrelationen Nr. 7, Nr. 9, Nr. 13, Nr. 14 und mit der in dieser Arbeit vorgeschlagenen Korrelation, wobei das gute Abschneiden der Korrelation von Dukler (Nr. 7) auf dem hohen Anteil von Messdaten mit niedrigem Dampfgehalt (vgl. Bild 7) beruht.

Berücksichtigt man die auch bei Verwendung der Korrelation von Bandel [5] verbleibende Unsicherheit sowie die zwischen den Messdaten auftretenden Abweichungen, dann kann die in dieser Arbeit vorgeschlagene, wesentlich einfachere Korrelation in den meisten Fallen empfohlen werden.

1. Introduction

Since frictional pressure drop for two-phase gas–liquid flow is an important parameter for the design of pipelines and evaporators, numerous investigations on this topic can be found in the literature. Although it is still not possible to predict theoretically the mechanisms occurring in two-phase flow, a considerable number of empirical correlations for the prediction of frictional pressure drop exist. For conditions outside the range of the original data from which these correlations were derived, however, deviations of several 100% between predicted and measured values may be found [1].

Consequently, data banks, which contain measurements with a number of liquid–gas combinations for various flow conditions and pipe diameters, have been assembled [2–4]. Correlations fitted to these data banks, however, often have the disadvantage of containing a large number of constants and of being inconvenient to use. An example is the correlation developed by Bandel [5], which is given in full in Appendix B.

In what follows, a particularly simple correlation for the prediction of frictional two-phase pressure drops is developed which gives satisfactory agreement with measured data.

The predictions of this correlation, and of fourteen correlations suggested by other authors, will be compared with measured data in order to find out which correlations are suitable for the prediction of frictional pressure drop.

2. Description of the new correlation

Figure 1 shows the frictional pressure drop as a function of the flow quality. Owing to the increasing interaction between the gas and liquid phases, the frictional pressure drop increases with increasing flow quality, passes through a maximum for $\dot{x} \cong 0.85$, and then falls to the frictional pressure drop for single-phase gas flow for $\dot{x} = 1$. W. Bonn (Fachhochschule Rüsselsheim, Rüsselsheim, F.R.G.) observed for his measurements with nitrogen that the two-phase pressure drop for $\dot{x} = 0.5$ is nearly always identical to the single-phase gas pressure drop for $\dot{x} = 1$. Using the pressure drop of the respective single-phase flow

$$\left(\frac{dp}{dL}\right)_{f,\ell} = \zeta_\ell \frac{\dot{m}^2}{2\rho_\ell d} = A \qquad (1)$$

$$\left(\frac{dp}{dL}\right)_{f,g} = \zeta_g \frac{\dot{m}^2}{2\rho_g d} = B \qquad (2)$$

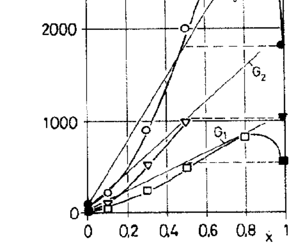

Fig. 1. Influence of flow quality on frictional pressure drop of R12, according to ref. 6.

Bild 1. Einfluss des Dampfgehaltes auf den Reibungsdruckverlust von R12, nach Lit. 6.

with

$$\zeta_\ell = \frac{64}{Re_\ell}, \quad \zeta_g = \frac{64}{Re_g} \quad \text{for} \quad Re_\ell, Re_g \leqslant 1187 \tag{3}$$

$$\zeta_\ell = \frac{0.3164}{Re_\ell^{1/4}}, \quad \zeta_g = \frac{0.3164}{Re_g^{1/4}} \quad \text{for} \quad Re_\ell, Re_g > 1187 \tag{4}$$

and

$$Re_\ell = \frac{\dot{m}d}{\eta_\ell}, \quad Re_g = \frac{\dot{m}d}{\eta_g} \tag{5}$$

an equation for the roughly linear increase of the pressure drop with increasing quality for $\dot{x} < 0.7$ can be written:

$$G = A + 2(B - A)\dot{x} \tag{6}$$

To cover the full range of flow quality $0 \leqslant \dot{x} < 1$, a superimposition of eqns. (2) and (6) was used:

$$\left(\frac{dp}{dL}\right)_{f,\,tp} = G(1 - \dot{x})^{1/C} + B\dot{x}^C \tag{7}$$

A value of $C = 3$ was found by curve fitting measured data.

In addition to its simplicity, eqn. (7) has the advantage of being easily integrated if the flow quality increases along the evaporator tube due to heating with constant heat flux. Thus

$$\int_0^L \left(\frac{dp}{dL}\right)_{f,\,tp} dL = \left\{-\frac{3}{4}(1 - \dot{x})^{4/3}[A + 2(B - A)\dot{x}] + \frac{1}{4}B\dot{x}^4 - \frac{9}{14}(B - A)(1 - x)^{7/3}\right\}_{\dot{x}_{in}}^{\dot{x}_{out}} \tag{8}$$

The exit flow quality is obtained from an energy balance

$$\frac{d\dot{x}}{dL} = \frac{4\dot{q}}{\dot{m}d\,\Delta h_v} \tag{9}$$

If other correlations are used, this integration has to be done numerically [7]. Equations (7) or (8) apply only for the prediction of frictional pressure drop. The static pressure drop has to be added if the flow direction is not horizontal, as does the acceleration pressure drop if evaporation occurs.

The application of eqn. (7) should be restricted to flow conditions where

$$Re_\ell = \dot{m}d/\eta_\ell > 100 \tag{10}$$

For lower mass velocities and for viscous liquids, the frictional pressure drop for $\dot{x} = 0.5$ may differ considerably from the value for $\dot{x} = 1$. Furthermore, eqn. (7) can be used only as long as the frictional pressure drop of the gas flow is higher than the frictional pressure drop of the liquid flow ($B > A$). For certain oil–gas flow rates, this condition may not be fulfilled. Nevertheless, it should be noted that not only eqn. (7), but all the correlations investigated in this paper, fail to predict reasonable values for the above conditions. In particular, the correlation developed by Friedel [2] gives values which differ from the measured data by several orders of magnitude.

3. Measured and predicted influence of parameters on frictional pressure drop

To check the performance of the proposed correlation, the predicted influence of parameters such as flow quality, mass velocity and system pressure is compared with the respective measurements in Figs. 2, 3 and 4. The data for the flow of air–water were taken from

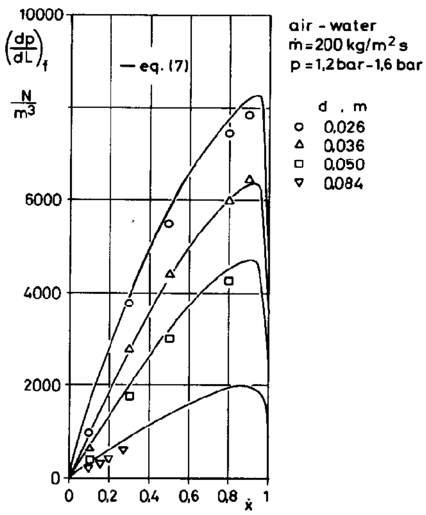

Fig. 2. Measured and calculated frictional pressure drop of water and air as a function of flow quality.

Bild 2. Gemessener und berechneter Einfluss des Dampfgehaltes auf den Reibungsdruckverlust von Wasser und Luft.

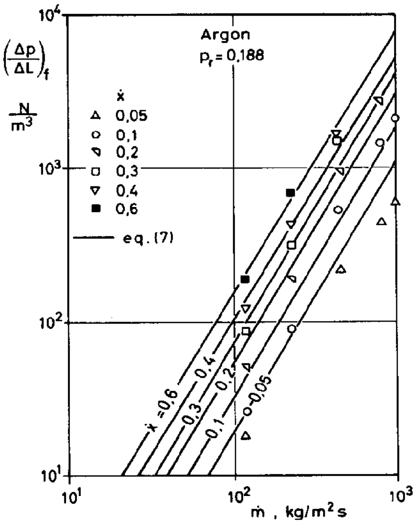

Fig. 3. Measured and calculated frictional pressure drop of argon as a function of mass velocity.

Bild 3. Gemessener und berechneter Einfluss der Massenstromdichte auf den Reibungsdruckverlust von Argon.

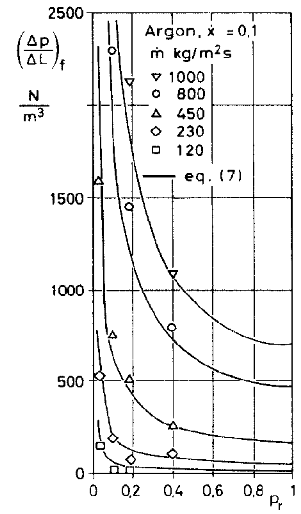

Fig. 4. Measured and calculated frictional pressure drop of argon as a function of reduced pressure.

Bild 4. Gemessener und berechneter Einfluss des reduzierten Druckes auf den Reibungsverlust von Argon.

Reza-Chavez [8] and the data for flow of argon from Müller-Steinhagen [1]. For the relatively long test sections used by Reza-Chavez, the pressure drop considerably reduced the total system pressure and therefore the gas density. This caused the non-linear increase in the measured and predicted pressure drop shown in Fig. 2 for $\dot{x} < 80\%$. The investigations with nitrogen and argon [9, 1] were performed at elevated pressure using short test sections ($0.175\ \mathrm{m} \leqslant L \leqslant 0.525\ \mathrm{m}$) and consequently the decrease in system pressure with increasing pressure drop may be neglected.

Figure 2 shows that eqn. (7) gives a satisfactory prediction of the influence of flow quality and pipe diameter. The same influence of mass velocity on frictional pressure drop is predicted by eqn. (7) for both single- and two-phase flow. As shown in Fig. 3, this prediction gives realistic results. Because the slip between gas flow and liquid flow is reduced with increasing system pressure (owing to the increasing gas density), the frictional pressure drop for constant flow quality and mass velocity also decreases with increasing system pressure. This trend is also predicted by eqn. (7).

4. Comparison of correlations

The performance of eqn. (7) was compared with trends predicted by the correlations given in Table 1. Correlation No. 13 in this Table is a further development of eqn. (7), recently suggested by Reza-Chavez [8, 20]. Reza-Chavez uses the expression

$$x_0 = 1.6\,(\eta_g/\eta_\ell)^{0.15}\,[1 + \log(\mathrm{Re}_\ell\,\mathrm{Fr}_\ell)]^{-1/3} \qquad (11)$$

with

$$x_0 = 1 \text{ if } \dot{m} < 25\ \mathrm{kg\,m^{-2}\,s^{-1}} \quad \text{or if} \quad x_0 > 1 \qquad (12)$$

TABLE 1. Correlations used for comparison (Zum Vergleich verwendete Korrelationen)

No.	Author	Ref.
1	Bandel	[5]
2	Bankoff	[10]
3	Chawla	[11]
4	Chawla–Bankoff	[12]
5	Chisholm–Baroczy	[13]
6	Cicchitti	[14]
7	Dukler	[15]
8	Friedel	[2]
9	Gronnerud	[16]
10	Kesper–Moussalli	[17]
11	Lockhart–Martinelli	[18]
12	Lombardi–Pedrocchi	[19]
13	Reza-Chavez	[8]
14	Storek–Brauer	[3]
15	Eqn. (7)	[1]

to predict the flow quality for which the two-phase pressure drop equals the single-phase gas pressure drop. However, for a number of flow conditions this expression could not be solved since $\log(\mathrm{Re}_\ell\,\mathrm{Fr}_\ell)$ was less than -1. The limiting condition (12) was then tentatively changed to

$$x_0 = 1 \text{ if } \mathrm{Re}_\ell \mathrm{Fr}_\ell \leqslant 0.1 \quad \text{or if} \quad x_0 > 1 \qquad (13)$$

a procedure that caused only negligible loss of accuracy if the correlation was compared with data originally used by Reza-Chavez. After this minor modification, no more problems occurred within the flow conditions covered in this paper.

Some authors [2, 3] have suggested different correlations for different flow directions, for example, horizontal, vertically upwards and vertically downwards. Calculations for this paper were then done with the appropriate equations.

For the various correlations, Figs. 5(a) and (b) show measured and predicted values of frictional pressure drop as a function of flow quality. The measured data for nitrogen were taken from Bonn [9]. Numbers in the diagrams refer to the correlations given in Table 1.

While most of the predicted trends agree fairly well with the measurements, Fig. 5 indicates that correlations 2, 3, 4 and 12 tend to overpredict the pressure drop considerably. Owing to the terms $(1-\dot{x})/\dot{x}$ or $\dot{x}/(1-\dot{x})$, single-phase gas or liquid pressure drop is not included in correlations 1, 2, 3, 4, 10, 11 and 14. Furthermore, the correlation by Bandel (No. 1), as it is given in Appendix B, can be used only for $\dot{x} < 0.98$ while at higher values of flow quality the square root of negative values may occur.

The correlations suggested by Cicchitti and Dukler (Nos. 6 and 7) have been developed using a homogeneous flow model. Therefore, both correlations tend to underpredict the frictional pressure drop for higher flow qualities.

Figure 6 is a plot of the frictional pressure drop for two-phase flow of argon versus the flow quality for

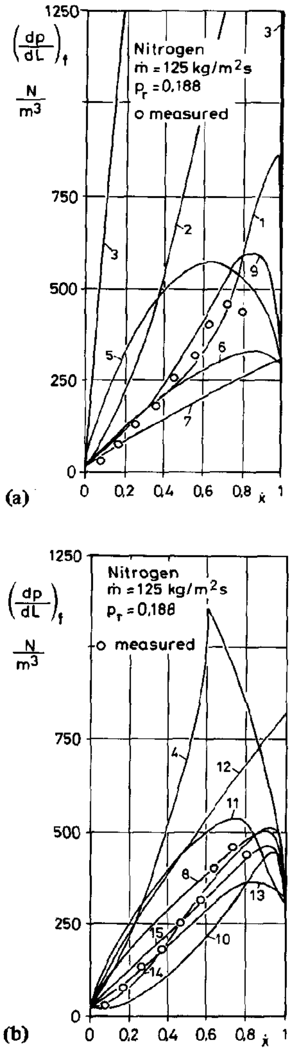

Fig. 5. Influence of flow quality on frictional pressure drop of nitrogen, calculated according to correlations given in Table 1.

Bild 5. Gemessener und nach verschiedenen Korrelationen berechneter Reibungsdruckverlust von Stickstoff als Funktion des Dampfgehaltes.

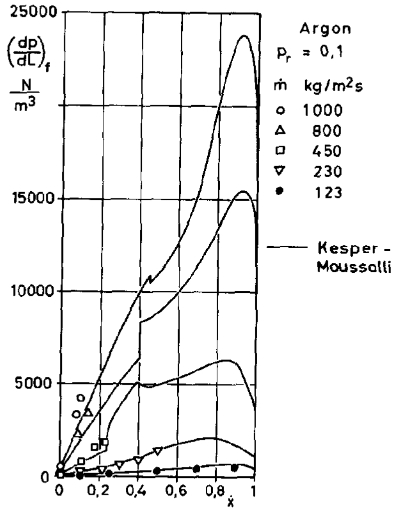

Fig. 6. Frictional pressure drop vs. flow quality, calculated according to ref. 17.

Bild 6. Einfluss des Dampfgehaltes auf den Reibungsdruckverlust, berechnet nach Lit. 17.

various mass velocities. The curves were calculated using a combination of correlations by Kesper [21] and Moussalli [22], as is suggested by Chawla in the *VDI-Wärmeatlas* [17]. Although measured and predicted values agree quite well, irregular shapes which may not exist in reality are found in the predicted curves for certain flow conditions.

The simple correlation suggested earlier in this paper includes the frictional pressure drop for single-phase liquid or gas flow ($\dot{x} = 0$ or $\dot{x} = 1$) and can be applied over the full range of flow quality, $0 \leqslant \dot{x} \leqslant 1$.

5. Comparison of measured and predicted values

To allow quantitative assessment of the correlations given in Table 1, all correlations were checked against a data bank with measurements of frictional pressure drop. Details on contents and composition of the data bank are given in Appendix A. While a total of 9313 data sets could be used for comparison with correlations 5, 6, 7, 8, 9, 12, 13, 15, only 9256 data could be used for correlations 2, 3, 4, 10, 11, 14 and only 8541 for correlation 1, because of the restrictions mentioned before.

As characteristic values for the performance of the respective correlations, the average relative error

$$\mathrm{RE} = \frac{1}{n} \sum^{n} \left| \frac{(\mathrm{d}p/\mathrm{d}L)_{\mathrm{meas}} - (\mathrm{d}p/\mathrm{d}L)_{\mathrm{pred}}}{(\mathrm{d}p/\mathrm{d}L)_{\mathrm{meas}}} \right| \tag{14}$$

and the average absolute error

$$\mathrm{AE} = \frac{1}{n} \sum^{n} |(\mathrm{d}p/\mathrm{d}L)_{\mathrm{meas}} - (\mathrm{d}p/\mathrm{d}L)_{\mathrm{pred}}| \tag{15}$$

were used as well as the percentage of predicted values within ±10%, ±20% and ±30% of the measured data. In addition, comparison between measured and predicted trends (see §3) should be used for a final judgement concerning the reliability of correlations. This is especially necessary since the composition of the data bank is not homogeneous.

Table 2 gives values of RE and AE for the correlations listed in Table 1. Since no systematic deviations were found for different flow directions, Table 2 is based on all data mentioned above. Obviously, the correlation by Bandel (No. 1) gives the best agreement between predicted and measured values. Therefore, this correlation is given in full in Appendix B. Considering the scatter of data, reasonable accuracy is also obtained using the correlations suggested by Gronnerud (No. 9), Reza-Chavez (No. 13), and Storek–Brauer (No. 14),

TABLE 2. Comparison between predicted and measured values of frictional pressure drop (Übereinstimmung zwischen berechneten und gemessenen Reibungsdruckverlusten)

No.	Author	RE (%)	AE ($N m^{-3}$)	RE < 10% (%)	RE < 20% (%)	RE < 30% (%)
1	Bandel	32.6	5347.1	25.8	44.7	59.9
2	Bankoff	11525.8	139561.5	7.5	12.6	16.4
3	Chawla	8697.6	696831.7	5.5	10.5	15.3
4	Chawla–Bankoff	142.3	47471.3	18.1	32.6	40.9
5	Chisholm–Baroczy	340.0	5579.6	16.5	28.9	38.2
6	Cicchitti	65.7	4531.7	15.8	30.0	42.0
7	Dukler	37.0	4916.6	14.7	29.0	43.9
8	Friedel	111.6	5015.0	18.1	32.6	44.6
9	Gronnerud	44.6	10808.0	16.0	31.4	46.5
10	Kesper–Moussalli	69.9	6920.6	12.5	22.3	29.9
11	Lockhart–Martinelli	62.8	14244.2	21.0	38.0	52.4
12	Lombardi–Pedrocchi	152.3	4910.4	14.2	22.2	29.5
13	Reza-Chavez	35.5	5490.5	18.1	37.4	54.6
14	Storek–Brauer	36.5	7859.0	22.2	41.9	58.7
15	Eqn. (7)	41.9	5481.4	17.3	34.5	49.5

as well as using the correlation suggested in this paper. The correlation by Reza-Chavez is particularly useful for flow of air/water and air/water/carboxyl methyl cellulose (CMC). Although results obtained with correlation Nos. 6 and 7 are quite respectable, application of these equations should be limited to lower flow qualities since the homogeneous model generally fails for high quality. Values predicted by the well-known Lockhart–Martinelli model were usually higher than the measured values.

Recently, the correlations by Friedel (No. 8) and by Storek and Brauer (No. 14) have been particularly recommended. Therefore, the performance of these two correlations and that of the correlation suggested in this paper are given separately in Fig. 7 for flows of water, refrigerant R12 and argon. The data for this Figure are given in Table 3. With respect to the average absolute error and the percentage of data within a certain error range, Friedel's correlation has reasonable results. The average relative error, however, is clearly higher than for a number of other correlations. As Figs. 5 and 7 show, this result is mainly caused by the poor accuracy for lower flow quality (low frictional pressure drop) where this correlation considerably overpredicts the pressure drop. In addition, the correlation can fail totally for fluids with a high viscosity ratio η_ℓ/η_g. The performance of the correlation suggested by Storek and Brauer is clearly superior. Nevertheless, it should be mentioned that it may fail for high mass flow rates and flow qualities, where predicted values may exceed the measured data considerably. The simple correlation suggested in this paper gives reasonable results for all flow conditions. Predicted values generally lie between those predicted by the above correlations.

6. Conclusions

The simple correlation suggested in this paper, as well as fourteen correlations from the literature, have been checked against an extensive data bank with measurements of frictional pressure drop in pipes. Best agreement between predicted and measured values was obtained with the correlation by Bandel [5] which, however, is quite lengthy and complicated to use. The correlation suggested in this paper is more convenient, and still predicts the frictional pressure drop with reasonable accuracy. It includes single-phase liquid and gas pressure drop and predicts correctly the influence of flow parameters.

Nevertheless, it has to be concluded from the above investigation that the prediction of frictional pressure drop for two-phase flow in pipes is far from satisfactory. Average deviations of more than ±30% between predicted and measured values still have to be accepted.

Acknowledgements

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TABLE 3. Data for Figs. 7(a)–(c) (Angaben zu den Bildern 7(a)–(c))

Figure	7(a)	7(b)	7(c)
Fluid	Steam–water	Refrigerant R12	Argon
Author	[21]	[6]	[1]
Data	392	151	109
Tube diameter (m)	0.0243	0.014	0.014
Orientation	Vertical	Horizontal	Horizontal
Mass velocity ($kg m^{-2} s^{-1}$)	291–2490	50–246	66–1038
Flow quality	0.03–0.97	0.1–0.81	0.01–0.91
Density ratio ρ_ℓ/ρ_g	2.8–31.0	77.7–159.6	10.7–152.6
Viscosity ratio η_ℓ/η_g	1.9–5.8	23.1–30.4	7.5–29.8

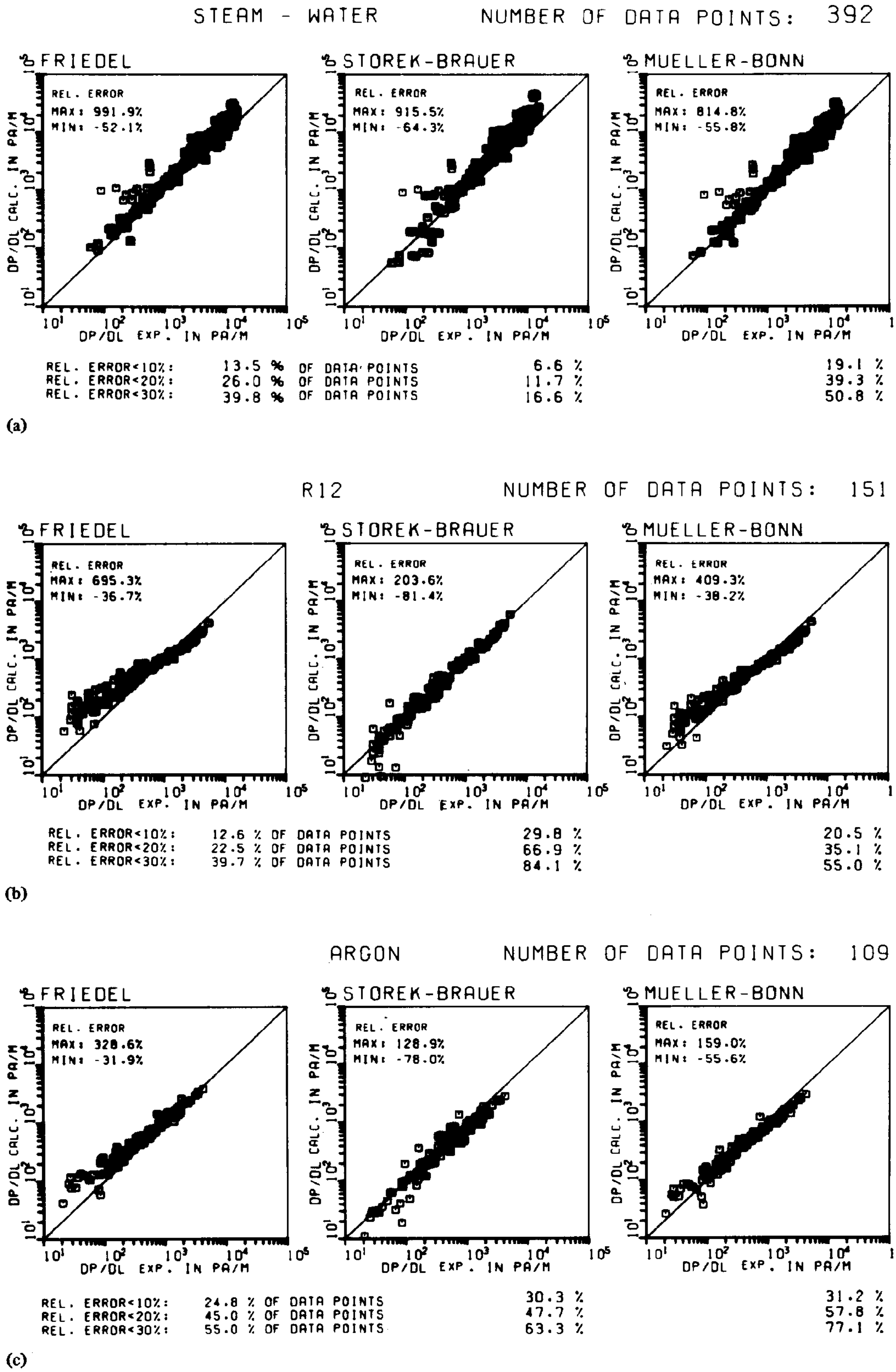

Fig. 7. Comparison between calculated and measured values of frictional pressure drop for water, R12 and argon.

Bild 7. Vergleich zwischen berechneten und gemessenen Reibungsdruckverlusten bei verschiedenen Fluiden.

Nomenclature

A	single-phase liquid pressure drop, N m^{-3}
AE	absolute error, N m^{-3}
B	single-phase gas pressure drop, N m^{-3}
C	constant
d	tube diameter, m
Fr	Froude number
g	acceleration due to gravity, m s^{-2}
Δh_v	latent heat of evaporation, J kg^{-1}
k	average roughness, m
k/d	relative roughness
L	length, m
$\dot{m}$	mass velocity, kg m^{-2} s^{-1}
p	pressure, bar
p_c	critical pressure, bar
p_r	$= p/p_c$, reduced pressure
$\dot{q}$	heat flux, W m^{-2}
Re	Reynolds number
RE	percentage average relative error
$\dot{x}$	$= \dot{m}_g/(\dot{m}_g + \dot{m}_\ell)$, flow quality
ζ	friction factor
η	viscosity, kg m^{-1} s^{-1}
ρ	density, kg m^{-3}
σ	surface tension, N m^{-1}

Indices

c	critical
calc	calculated
f	frictional
g	gas
in	inlet
ℓ	liquid
max	maximum
meas	measured
min	minimum
out	outlet
ph	phase
pred	predicted
Ri	annular flow
Sch	stratified flow
tp	two phase

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Appendix A

Description of the data bank

For comparison with the correlations given in Table 1, a data bank with 9313 measurements of frictional pressure drop was used. To avoid uncertainties due to the scatter of data, this data bank contains only measurements with $dp/dL > 20$ N m^{-3}. About two-thirds of the data were taken from Dukler's data bank [4]. A summary of these data is given in Table A-1. To keep the original data, this part of the data bank was stored in American units.

The second part of the data bank was put together in the Institut für Thermische Verfahrenstechnik (TVT),

TABLE A-1. Contents of Dukler's data bank (Inhalt der Dukler-Datenbank)

No.	Author	Data	Type[a]	Tube diam. (mm)	System
1	Wicks	112	LH	26	Air–water
2	Wicks	108	LH	26	Air–water
3	Dukler	220	LH	26	Air–water
4	Company A	417	FT	154	Hydrocarbons
5	Company B	59	FT	392	Hydrocarbons
6	Gazley	144	LH	52	Air–water
7	Melvin	70	LH	26	Air–oil
8	Jenkins	566	LH	25	Air–water
9	Fritzlen	90	LH	26	Air–water
10	Chenoweth	173	LH	40	Air–water
11	Chenoweth	102	LH	78	Air–water
12	Reid	15	LH	102	Air–water
13	Chenoweth	24	LH	154	Air–water
14	Cowan	68	LVD	26	Air–oil
15	King	21	LH	19	Air–water
16	Wickey	106	LH	12	Steam–water
17	Mosher	93	LH	27	Steam–water
18	Abou-Sabe	63	LH	22	Air–water
19	Alves	148	LH	27	Air–water
20	Alves	149	LH	27	Air–oil
21	Company A	1110	LH	50	Air–water
22	Company A	191	LH	91	Air–oil
23	Company A	155	LH	140	Air–oil
24	Company A	78	LH	49	Air–oil
25	Company A	79	LH	50	Air–oil
26	Company A	34	LH	48	Air–oil
27	Company A	41	LH	50	Air–oil
28	Chisholm	84	LH	27	Air–water
29	Chisholm	79	LH	26	Air–water
30	Chisholm	76	LH	27	Air–water
31	Chisholm	75	LH	27	Air–water
32	Chisholm	44	LH	26	Air–water
33	Chisholm	65	LH	26	Air–water
34	Moen	132	LH	12	Steam–water
35	Magiros	148	LH	26	Air–mod. water
36	Magiros	257	LH	26	Air–water
37	Silvestri	47	LVU	5	Steam–water
38	Silvestri	113	LVU	5	Steam–water
39	Silvestri	173	LVU	5	Steam–water
40	Silvestri	74	LVU	6	Steam–water
41	Silvestri	57	LVU	8	Steam–water
42	Silvestri	27	LVU	10	Steam–water
43	Silvestri	362	LVU	25	Argon–water
44	Silvestri	108	LVU	25	Argon–(water) ethanol
45	White	293	LH	41	Air–(kerosene, oil) water
46	White	383	LH	41	Gas–(kerosene, oil) water
47	Sher	355	LVU	22	Steam–water
48	Larson	31	LH	12	Steam–water

[a]LH, laboratory data, flow direction = horizontal; LVU, laboratory data, flow direction = vertically upwards; LVD, laboratory data, flow direction = vertically downwards; FT, field data, test conditions.

University of Karlsruhe, F.R.G., mostly from recent Ph.D. theses and publications. Table A-2 gives some information about these data, which were published and stored in SI units.

TABLE A-2. Contents of TVT data bank (Inhalt der TVT-Datenbank)

No.	Author	Data	Type[a]	Tube diam. (mm)	System
1	Chawla	26	LH	6.0	R11–R11
2	Chawla	35	LH	14.0	R11–R11
3	Chawla	27	LH	25.0	R11–R11
4	Bandel	22	LH	14.0	R11–R11
5	Bandel	35	LH	14.0	R12–R12
6	Bandel	31	LH	14.0	R22–R22
7	Müller	114	LH	14.0	Argon–argon
8	Bonn	72	LH	14.0	Nitrogen–nitrogen
9	Mohr	55	LH	6.0	Neon–neon
10	Mohr	41	LH	4.0	Neon–neon
11	Iwicki	158	LH	14.0	R12–R12
12	Reza-Chavez	15	LH	26.0	Air–water
13	Reza-Chavez	15	LH	36.0	Air–water
14	Reza-Chavez	14	LH	50.0	Air–water
15	Reza-Chavez	32	LH	84.0	Air–water
16	KWU	78	LVU	12.5	Steam–water
17	KWU	187	LH	12.5	Steam–water
18	KWU	392	LH	24.3	Steam–water
19	Reza-Chavez	109	LH	26.0	Air–water–CMC
20	Reza-Chavez	119	LH	36.0	Air–water–CMC
21	Reza-Chavez	126	LH	50.2	Air–water–CMC
22	Reza-Chavez	54	LH	84.2	Air–water–CMC
23	Reza-Chavez	100	LH	45.0	Air–water
24	Reza-Chavez	107	LH	45.0	Air–water
25	Reza-Chavez	93	LH	45.0	Air–water–CMC
26	Reza-Chavez	87	LH	45.0	Air–water–CMC
27	Janssen	14	LH	18.9	Steam–water
28	Janssen	14	LH	32.2	Steam–water
29	Janssen	7	LH	24.3	Steam–water
30	Janssen	8	LH	24.3	Steam–water
31	Janssen	22	LH	24.3	Steam–water
32	Takahashi	11	LH	201.3	Steam–water
33	Wairakei	83	FT	203.0	Steam–water
34	Wairakei	83	FT	203.0	Steam–water
35	Nguyen	269	LH	45.5	Air–water
36	McMillan	115	LH	46.7	R11–R11
37	Hewitt	15	LH	12.7	R12–R12
38	Hewitt	19	LH	16.0	R12–R12
39	Hewitt	31	LH	19.0	R12–R12
40	Hewitt	15	LH	25.8	R12–R12
41	Hewitt	21	LH	25.8	R12–R12
42	Hewitt	16	LH	25.8	R12–R12
43	Hewitt	58	LH	27.5	R12–R12
44	Hewitt	62	LH	35.4	R12–R12

[a]LH, laboratory data, flow direction = horizontal; LVU, laboratory data, flow direction = vertically upwards; FT, field data, test conditions.

Analysis of measured and predicted values shows that the composition of the data bank has considerable influence on the final result obtained for certain correlations. Values predicted with the correlation of Storek and Brauer, for example, agree quite well with the measured frictional pressure drop if mass velocity and flow quality are not too high. For high values of $\dot{m}$ and $\dot{x}$, this correlation tends to overpredict the pressure drop considerably. Another example is the correlation suggested by Friedel, which fails for fluids with high viscosity ratio η_ℓ/η_g.

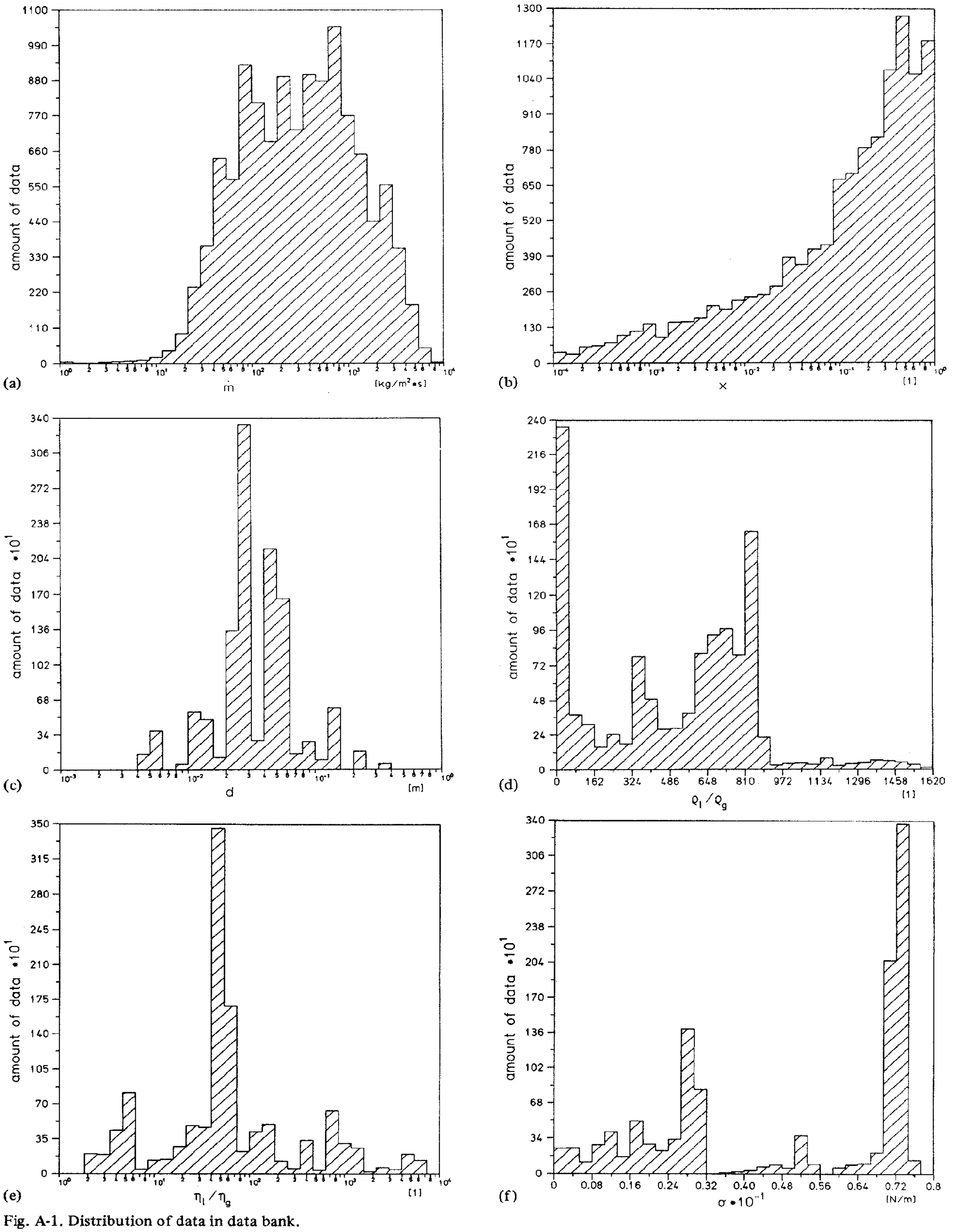

Fig. A-1. Distribution of data in data bank.

Bild A-1. Häufigkeitsverteilung der Messdaten in der Datenbank.

To check the performance of correlations, the distribution of influencing parameters in a data bank, therefore, should be as homogeneous as possible. Since this is generally not possible, information about the composition of the data bank should be provided, at least. Figures A-1(a)–(f) show the distribution of data with respect to mass velocity, flow quality, tube diameter, density ratio, viscosity ratio and surface tension. Although most technical applications may be covered by the range of parameters, most of the data are within a narrow span of physical properties. The fact that 50% of data are within $0 \leqslant \dot{x} \leqslant 0.1$ supports correlations with better accuracy for low quality, that is, correlations based on a homogeneous model. Therefore, more detailed analysis on the accuracy of correlations for specific fluids and flow conditions is desirable.

Appendix B

Prediction of frictional pressure drop according to Bandel [5]

$$\theta = \frac{\eta_\ell}{\eta_g}, \qquad f = \frac{d^2\pi}{4}, \qquad g = 9.8065$$

$$\mathrm{Re}_g = \frac{\dot{m}\dot{x}d}{\eta_g}, \qquad \mathrm{Re}_\ell = \frac{\dot{m}(1-\dot{x})d}{\eta_\ell}$$

$$\left(\frac{\Delta p}{\Delta L}\right)_g = \zeta_g \frac{\dot{m}^2\dot{x}^2}{2\rho_g d}, \qquad \zeta_g = \frac{0.3164}{\mathrm{Re}_g^{0.25}}$$

$$\left(\frac{\Delta p}{\Delta L}\right)_\ell = \zeta_\ell \frac{\dot{m}^2(1-\dot{x})^2}{2\rho_\ell d}, \qquad \zeta_\ell = \frac{0.3164}{\mathrm{Re}_\ell^{0.25}}$$

Calculation for various flow regimes

$$f_{\mathrm{ph,\,g,\,Ri}} = 0.15\left(\frac{1-\dot{x}}{\dot{x}}\right)^{0.5}\left(\frac{\eta_\ell}{\eta_g}\right)^{0.3} \qquad \text{if} \quad \dot{x} \leqslant 0.5$$

$$f_{\mathrm{ph,\,g,\,Ri}} = 0.16(1-\dot{x})^{0.1}\left(\frac{\eta_\ell}{\eta_g}\right)^{0.3} \qquad \text{if} \quad \dot{x} > 0.5$$

$$f_{\mathrm{ph,\,\ell,\,Ri}} = -0.31\,\dot{x}^{0.1}$$

$$\left(\frac{\Delta p}{\Delta L}\right)_{\mathrm{Sch,\,max}} = \frac{g\rho_\ell \dot{x}}{15(\eta_\ell/\eta_g)^{0.2}}$$

$$\left(\frac{\Delta p}{\Delta L}\right)_{\mathrm{Ri,\,min}} = \frac{g\rho_\ell}{10\dot{x}^{0.3}}$$

Annular flow

$$\mathrm{V1}_{\mathrm{Ri}} = \frac{(\Delta p/\Delta L)_g}{(\Delta p/\Delta L)_\ell}\,\frac{0.3164 + f_{\mathrm{ph,\,g,\,Ri}}}{0.3164 + f_{\mathrm{ph,\,\ell,\,Ri}}}$$

Initial values

$$\varphi = \frac{1}{500} - \frac{1}{10}, \qquad B = \frac{1}{10}$$

→ $\varphi = \varphi + B$

$$d_{\mathrm{g,\,Ri}} = (1-\varphi)d, \qquad d_{\ell,\,\mathrm{Ri}} = d - d_{\mathrm{g,\,Ri}}$$

$$f_{\mathrm{g,\,Ri}} = \frac{\pi d_{\mathrm{g,\,Ri}}^{\,2}}{4}, \qquad f_{\ell,\,\mathrm{Ri}} = f - f_{\mathrm{g,\,Ri}}$$

$$\text{if} \quad \frac{d_{\ell,\,\mathrm{Ri}}}{d} > 0.999 \longrightarrow \mathrm{I}$$

$$\mathrm{V2}_{\mathrm{Ri}} = \left(\frac{d_{\mathrm{g,\,Ri}}}{d_{\ell,\,\mathrm{Ri}}}\right)^{1.25}\left(\frac{f_{\mathrm{g,\,Ri}}}{f_{\ell,\,\mathrm{Ri}}}\right)^{1.75}$$

$$\text{if} \quad \frac{\mathrm{V2}_{\mathrm{Ri}} - \mathrm{V1}_{\mathrm{Ri}}}{\mathrm{V1}_{\mathrm{Ri}}} \leqslant \frac{1}{100} \longrightarrow \mathrm{I}$$

← if $\mathrm{V2}_{\mathrm{Ri}} > \mathrm{V1}_{\mathrm{Ri}}$, if $\mathrm{V2}_{\mathrm{Ri}} = \mathrm{V1}_{\mathrm{Ri}} \longrightarrow \mathrm{I}$

← if $\mathrm{V2}_{\mathrm{Ri}} < \mathrm{V1}_{\mathrm{Ri}}$: $\varphi = \varphi - B$, $B = B/2$

$$\mathrm{I} \quad \mathrm{CF}_{\mathrm{Ri}} = \left[1 + \left(\frac{f_{\mathrm{ph,\,g,\,Ri}}}{0.3164}\right)\left(\frac{d}{d_{\mathrm{g,\,Ri}}}\right)^{1.25}\right]\left(\frac{f}{f_{\mathrm{g,\,Ri}}}\right)^{1.75}$$

$$\mathrm{DP}_{\mathrm{Ri}} = (\Delta p/\Delta L)_g\,\mathrm{CF}_{\mathrm{Ri}}$$

$$\text{if} \quad \mathrm{DP}_{\mathrm{Ri}} \geqslant \left(\frac{\Delta p}{\Delta L}\right)_{\mathrm{Ri,\,min}}: \qquad \left(\frac{\Delta p}{\Delta L}\right)_{\mathrm{tp}} = \mathrm{DP}_{\mathrm{Ri}}$$

$$\text{if} \quad \mathrm{DP}_{\mathrm{Ri}} < \left(\frac{\Delta p}{\Delta L}\right)_{\mathrm{Ri,\,min}}: \qquad \text{Stratified flow}$$

Stratified flow

$$\left(\frac{\Delta p}{\Delta L}\right)_\ell = \frac{64}{\mathrm{Re}_\ell}\,\frac{\dot{m}^2(1-\dot{x})^2}{2\rho_\ell d} \qquad \text{if} \quad \mathrm{Re}_\ell < 2300$$

$$\mathrm{V1}_{\mathrm{Sch}} = \frac{(\Delta p/\Delta L)_g}{(\Delta p/\Delta L)_\ell}$$

Initial values

$$\varphi = \frac{\pi}{1000} - \frac{\pi}{5}, \qquad B = \frac{\pi}{5}$$

→ $\varphi = \varphi + B$

$$f_{\ell,\,\mathrm{Sch}} = \frac{d^2(\varphi - \sin\varphi)}{8}, \qquad f_{\mathrm{g,\,Sch}} = f - f_{\ell,\,\mathrm{Sch}}$$

$$U_{\ell,\,\mathrm{Sch}} = \frac{d\varphi}{2} + d\sin\frac{\varphi}{2}$$

$$U_{\mathrm{g,\,Sch}} = d\left(\pi - \frac{\varphi}{2} + \sin\frac{\varphi}{2}\right)$$

$$d_{\mathrm{g,\,Sch}} = 4\frac{f_{\mathrm{g,\,Sch}}}{U_{\mathrm{g,\,Sch}}}, \qquad d_{\ell,\,\mathrm{Sch}} = 4\frac{f_{\ell,\,\mathrm{Sch}}}{U_{\ell,\,\mathrm{Sch}}}$$

$$\mathrm{V2}_{\mathrm{Sch}} = \left(\frac{d_{\mathrm{g,\,Sch}}}{d_{\ell,\,\mathrm{Sch}}}\right)^{1.25}\left(\frac{f_{\mathrm{g,\,Sch}}}{f_{\ell,\,\mathrm{Sch}}}\right)^{1.75}$$

if $\frac{V2_{Sch} - V1_{Sch}}{V1_{Sch}} \leqslant \frac{1}{100} \longrightarrow$ II

if $V2_{Sch} > V1_{Sch}$, if $V2_{Sch} = V1_{Sch} \longrightarrow$ II

if $V2_{Sch} < V1_{Sch}$: $\varphi = \varphi - B$, $B = B/2$

$$\text{II} \quad CF_{Sch} = \left(\frac{d}{d_{g,\,Sch}}\right)^{1.25} \left(\frac{f}{f_{g,\,Sch}}\right)^{1.75}$$

$$DP_{Sch} = (\Delta p/\Delta L)_g \, CF_{Sch}$$

if $DP_{Sch} \leqslant \left(\frac{\Delta p}{\Delta L}\right)_{Sch,\,max}$: $\underline{\left(\frac{\Delta p}{\Delta L}\right)_{tp} = DP_{Sch}}$

if $DP_{Sch} > \left(\frac{\Delta p}{\Delta L}\right)_{Sch,\,max}$: Transition region

Transition region

$$FH = \frac{0.3164\, \dot{x}^{1.75} \eta_g^{0.25}}{2 \rho_g d^{1.25}}$$

$$\dot{m}_{Ri,\,min} = \left(\frac{(\Delta p/\Delta L)_{Ri,\,min}}{CF_{Ri}\; FH}\right)^{1/1.75}$$

$$\dot{m}_{Sch,\,max} = \left(\frac{(\Delta p/\Delta L)_{Sch,\,max}}{CF_{Ri}\; FH}\right)^{1/1.75}$$

$$\underline{\left(\frac{\Delta p}{\Delta L}\right)_{tp}} = \left(\frac{\Delta p}{\Delta L}\right)_{Sch,\,max} \times \exp\left[\frac{\ln \dfrac{\dot{m}}{\dot{m}_{Sch,\,max}} \ln \dfrac{(\Delta p/\Delta L)_{Ri,\,min}}{(\Delta p/\Delta L)_{Sch,\,max}}}{\ln \dfrac{\dot{m}_{Ri,\,min}}{\dot{m}_{Sch,\,max}}}\right]$$

If relative roughness $k/d \geqslant 0.001$:

$$\underline{\left(\frac{\Delta p}{\Delta L}\right)_{tp} = \left(\frac{\Delta p}{\Delta L}\right)_{tp} \left(1000 \frac{k}{d}\right)^{0.25}}$$